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STABILITY OF THERMAL VIBRATIONAL FLOW IN AN INCLINED LIQUID LAYER AGAINST FINITE-FREQUENCY VIBRATIONS

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Stability of the flow that arises under the action of a gravity force and streamwise finite-frequency vibrations in a nonuniformly heated inclined liquid layer is studied. By the Floquet method, linearized convection equations in the Boussinesq approximation are analyzed. Stability of the flow against planar, spiral, and three-dimensional perturbations is examined. It is shown that, at finite frequencies, there are parametric-instability regions induced by planar perturbations. Depending on their amplitude and frequency, vibrations may either stabilize the unstable ground state or destabilize the liquid flow. The stability boundary for spiral perturbations is independent of vibration amplitude and frequency.

Key words: *thermal vibrational convection, parametric instability, inclined liquid layer.*

Introduction. Thermal vibrational convection has been the subject of many works, which were reviewed in [1]. The interest in this problem is justified both from practical and theoretical points of view. A periodical action, such as vibrations, exerts a strong influence on flow stability in static fields and can be used to control liquid flows in various technological processes, for instance, in production of high-purity materials on board sky-labs. In many works devoted to the study of the vibrational-convection problem, the case of small amplitudes and high frequencies was addressed, and the effective averaging method was used [2]. In the high-frequency limit, owing to the fact that the vibration period normally is short compared to characteristic hydrodynamic and thermal times of the system, the vibration amplitude and frequency are not independent parameters, and resonance effects are not observed.

The study of thermal vibrational convection under finite vibration frequencies is also of considerable interest. Tests performed on board sky-labs provide evidence for the g-effect (jitter), when the acceleration field involves a constant and a stationary component [3]. In many cases, the vibration amplitude and frequency are independent parameters. In such situations, instability mechanisms related to parametric resonance are manifested. Various aspects of convection in initially stationary liquids under the action of vertical finite-frequency vibrations representing gravity-force field modulations were examined in [4, 5]. Stability of thermovibrational flow that arises in a horizontal liquid layer under the action of longitudinal vibrations of an arbitrary frequency against planar perturbations was considered in [6] for zero-gravity conditions and in [7] for a static gravity field. The limiting cases of low and high vibration frequencies were examined, and parametric-instability regions for finite frequencies of the external action were determined.

However, in a gravity field, three-dimensional perturbations of thermovibrational flow, whose role was not discussed in [7], may be even more dangerous than planar perturbations. The competition between planar and three-dimensional instability modes in stationary convective flows was considered in [8]. During tilting of the horizontal layer, stratification of the nonuniformly heated liquid gives rise to a thermogravitational flow, which may have a profound effect on convective stability in the vibrational field. The mutual influence of the thermogravitational and

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vibrational instability mechanisms was previously considered for streamwise vibrations of the vertical layer only in the high-frequency limit [9].

In the present work, convective stability of a nonstationary flow of a uniform incompressible liquid in a flat layer under streamwise harmonic vibrations of finite frequency and amplitude is examined for arbitrary orientations of the layer relative to the gravity force. For a periodical nonisothermal flow, an analog of the Squire transformations, which make it possible to reduce the problem of stability against three-dimensional perturbations to a corresponding two-dimensional problem, is obtained. The behavior of planar, spiral, and three-dimensional perturbations is examined. It is shown that a static gravity field has a region of parameters in which stability of the flow both in the horizontal and inclined layers is governed by spiral perturbations.

1. Problem Statement. Planar Perturbations. We consider a flat layer of a uniform liquid whose boundaries are heated to different temperatures ($T = \mp\Theta$) and form an angle α_0 with the vertical direction. The whole layer is in a static field of a gravity force $\boldsymbol{g}_0$ and subjected to linear harmonic vibrations in the direction of the axis x parallel to its boundaries. The coordinates of the rigid planes restricting the flow are $z = \pm h$. The case where vibrational velocities are much lower than acoustic ones is analogous to the case with modulation of the gravity field according to the law $\boldsymbol{g} = \boldsymbol{g}_0 + b\Omega^2\boldsymbol{n}\sin(\Omega t)$, where $\boldsymbol{n} = (1, 0, 0)$ is the unit vector along the vibration axis, Ω is the angular frequency, and b is the displacement amplitude.

Using the quantity h as a distance scale, h^2/ν as a time scale, ν/h as a velocity scale, Θ as a temperature scale, and $\rho\nu^2/h^2$ as a pressure scale (ρ is the density of the liquid and ν is the kinematic viscosity), we write the convection equations in the Boussinesq approximation in the oscillating coordinate system:

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\nabla)\boldsymbol{v} = -\nabla p + \Delta\boldsymbol{v} + \mathrm{Gr}_{\mathrm{vib}} T\boldsymbol{n}\sin(\omega t) + \mathrm{Gr}T\cdot\boldsymbol{m},$$

$$\frac{\partial T}{\partial t} + \boldsymbol{v}\nabla T = \frac{1}{\mathrm{Pr}}\,\Delta T, \qquad \operatorname{div}\boldsymbol{v} = 0, \qquad \boldsymbol{m} = (\cos\alpha_0, 0, \sin\alpha_0), \tag{1}$$

$$z = \pm 1: \qquad \boldsymbol{v} = 0, \quad T = \mp 1.$$

Here $\boldsymbol{v}$ is the velocity, p is the pressure, T is the temperature counted from a certain mean value, $\mathrm{Gr} = g_0\beta_T\Theta h^3/\nu^2$ is the Grashof number, $\mathrm{Gr}_{\mathrm{vib}} = b\Omega^2\beta_T\Theta h^3/\nu^2$ is the vibrational analog of the Grashof number, β_T is the thermal-expansion coefficient, $\mathrm{Pr} = \nu/\chi$ is the Prandtl number, χ is the thermal diffusivity of the liquid, and $\omega = \Omega h^2/\nu$ is the dimensionless vibration frequency.

We can obtain an exact solution of system (1) in which the temperature field depends only on the cross-flow coordinate [$T_0 = T_0(z)$] and generates a periodical plane-parallel flow with a nonzero streamwise velocity $\boldsymbol{v}_0(v_0(z,t), 0, 0)$. This solution satisfies the boundary conditions

$$z = \pm 1: \qquad v_0 = 0, \quad T_0 = \mp 1 \tag{2}$$

and the flow-closure requirement

$$\int_{-1}^{1} v_0\,dz = 0. \tag{3}$$

From (1)–(3), we can find the velocity and temperature distributions in the ground state:

$$v_0 = (\mathrm{Gr}/6)(z^3 - z)\cos\alpha_0 + \mathrm{Gr}_{\mathrm{vib}}V_0(\omega, z, t) = \mathrm{Gr}f_1(z)\cos\alpha_0 + \mathrm{Gr}_{\mathrm{vib}}f_2(\omega, z), \quad T_0 = -z. \tag{4}$$

Thus, in the layer there arises a combined two-component flow with a velocity v_0. The first (thermogravitational) component with a cubic velocity profile is established in the layer even under no-vibration conditions. The intensity of this flow depends on the inclination angle of the layer: this intensity is maximal if the layer is oriented vertically ($\alpha_0 = 0$) and equals zero if $\alpha_0 = 90°$. The velocity profile of the second (thermovibrational) component depends on the vibration frequency and amplitude [6]:

$$V_0 = V_c(z,\omega)\cos(\omega t) + V_s(z,\omega)\sin(\omega t),$$

$$V_c = \frac{1}{\omega}\Big(z + \frac{\cosh\alpha\cos\beta - \cosh\beta\cos\alpha}{\cos 2æ - \cosh 2æ}\Big), \quad V_s = \frac{1}{\omega}\,\frac{\sinh\alpha\sin\beta - \sinh\beta\sin\alpha}{\cos 2æ - \cosh 2æ}, \tag{5}$$

$$\alpha = æ(1 + z), \quad \beta = æ(1 - z), \quad æ = \sqrt{\omega/2}.$$

Let us consider weak planar ground-state perturbations (5) $\boldsymbol{v}'(v'_x, 0, v'_z)$ $[v'_x = v'_x(x,z,t),\ v'_z = v'_z(x,z,t)]$, $T'(x,z,t)$, and $p'(x,z,t)$. Substituting the disturbed fields into the initial system (1) and performing linearization near the ground state, we obtain a system of equations for the perturbations. We introduce the stream function Ψ' for velocity perturbations

$$v'_x = -\frac{\partial \Psi'}{\partial z}, \qquad v'_z = \frac{\partial \Psi'}{\partial x} \tag{6}$$

and consider disturbances of the "normal"-mode type:

$$\Psi'(x,z,t) = \varphi(z,t)\exp(ikx), \qquad T'(x,z,t) = \theta(z,t)\exp(ikx). \tag{7}$$

Here φ and θ are the amplitudes and k is the wavenumber that describes the spatial periodicity of the perturbations. Substituting (6) and (7) into the equations for the perturbations and routinely eliminating the pressure, we obtain the following system of amplitude equations:

$$\begin{gathered}\frac{\partial \Delta\varphi}{\partial t} + ik\hat{H}(v_0)\varphi = \Delta^2\varphi + \mathrm{Gr}(ik\theta\sin\alpha_0 - \theta'\cos\alpha_0) - \mathrm{Gr}_{\mathrm{vib}}\theta'\sin(\omega t),\\ \frac{\partial\theta}{\partial t} - ik\varphi + ikv_0\theta = \frac{1}{\mathrm{Pr}}\,\Delta\theta, \quad \Delta \equiv \frac{d^2}{dz^2} - k^2, \quad \hat{H}(v_0)\varphi \equiv v_0\Delta\varphi - \varphi v_0'', \\ z = \pm 1: \qquad \varphi = \varphi' = 0, \quad \theta = 0.\end{gathered} \tag{8}$$

Hereinafter, the prime denotes differentiation with respect to the transverse coordinate z.

The amplitude problem (8) determines the behavior of "normal" perturbations. In the general case, to determine the stability limit for an arbitrary vibration amplitude and arbitrary vibration frequency, one has to use the Floquet theory [10]; with the help of this theory, existence conditions for periodical solutions of the amplitude problem can be found. To approximate the stream-function and temperature perturbations, we use sets of three-dimensional basis functions with time-dependent coefficients

$$\varphi = \sum_{m=0}^{M-1} a_m(t)\varphi_m, \qquad \theta = \sum_{m=0}^{M-1} b_m(t)\theta_m, \tag{9}$$

where M is the total number of the basis functions. As the basis functions φ_m and θ_m, we used the eigenfunctions of the problem of decaying perturbations in a stationary layer [8]. Inserting series expansions (9) into system (8) and performing orthogonalization by the Galerkin method, we obtain a set of ordinary differential equations for the coefficients $a_m(t)$ and $b_m(t)$, which can be integrated by the Runge–Kutta method. For arbitrary values of problem parameters, the solution of the problem of perturbations may be either growing in value or decaying. The stability boundary corresponds to periodical perturbations: subharmonic ones with a period twice longer than the period of the external action or synchronous ones whose period coincides with that of the vibrations.

All computations were performed for the unit Prandtl number $\mathrm{Pr} = 1$, for which the thermogravitational flow in the static field of the gravity force is unstable against the monotonic hydrodynamic mode. Hence, without vibrations, the perturbations at the stability boundary are stationary — the characteristic frequency of neutral oscillations ω_0 vanishes. This results in the fact that no quasi-periodic (two-frequency) perturbations occur if the flow experiences a periodic external action of frequency ω. For the majority of the solutions found, we used 16 basis functions ($M = 8$). In test computations performed with 20 basis functions ($M = 10$), the convection threshold varied by no more than 1%.

The stability boundaries on the plane $(\mathrm{Gr}_{\mathrm{vib}}, \mathrm{Gr})$ as functions of the inclination angle α_0 of the layer are shown in Fig. 1 for various modulation frequencies. The stability regions are bounded by the curves $\mathrm{Gr} = f(\mathrm{Gr}_{\mathrm{vib}})$ and by the coordinate axes. It should be noted that, as in the case of a horizontal layer [7], only synchronous-response perturbations were found in the examined range of parameters. Depending on the inclination angle of the layer and also on the vibration amplitude and frequency, the interplay of the vibrational and thermogravitational instability mechanisms may result either in stabilization or destabilization of the ground state. The limiting case $\mathrm{Gr} \to 0$ refers to a developed vibrational flow influenced by a weak thermogravitational flow. This influence is ambiguous: for inclination angles of the layer to the vertical larger than a certain critical angle $\alpha_*(\omega)$, the vibrational-convection threshold decreases with increasing Grashof number Gr, whereas in the case of $\alpha_0 < \alpha_{0*}(\omega)$, the vibrational flow becomes more stable. The limit $\mathrm{Gr}_{\mathrm{vib}} \to 0$ refers to weak vibrations influencing a developed thermogravitational flow. Depending on the inclination angle of the layer and frequency, either suppression of instability, for instance,

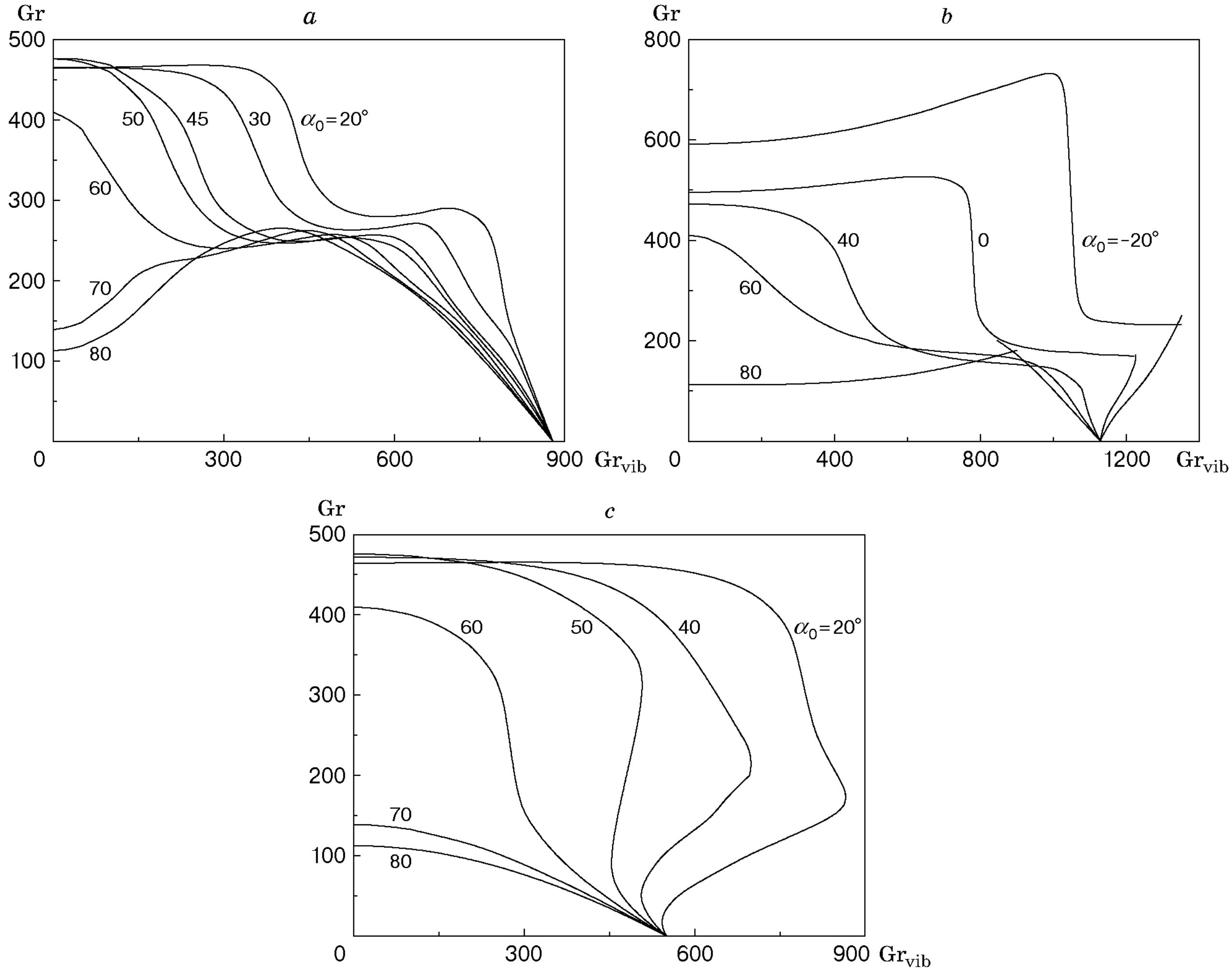

Fig. 1. Critical Grashof number Gr versus $\mathrm{Gr}_{\mathrm{vib}}$ for various inclination angles of the layer and $\omega = 2\pi$ (a), 4π (b), and 6π (c).

at $\alpha_0 = 70$ and $80°$ and $\omega = 2\pi$ (Fig. 1a), or a decrease in the threshold value of Gr for growing perturbations is possible.

In the range of $90° < \alpha_0 < 50°$, the position of the stability region in the case of strong vibrations $[\mathrm{Gr}_{\mathrm{vib}} > \mathrm{Gr}_{v*}(\omega)]$ changes insignificantly. Such a situation is observed when the intensity of the thermogravitational component of the main flow is much lower than the intensity of the thermovibrational component. For instance, at $\omega = 2\pi$ (Fig. 1a), the threshold Grashof number is $\mathrm{Gr}_{v*} \approx 250$. An increase in the vibration frequency results in a displacement of the thermovibrational-flow region toward the layer boundaries, thus narrowing the range of vibration amplitudes under which the thermogravitational flow only weakly affects the thermovibrational component. In the high-frequency limit, mutual influence of the instability mechanisms for the thermogravitational flow and the stratified liquid in the high-frequency vibrational field is observed.

At $\omega = 4\pi$ (Fig. 1b), the stability boundary for some inclination angles ($\alpha_0 = 80$, 0, and $-20°$) consists of two segments; near the intersection point of these segments, perturbations with different spatial periods compete with each other. At sufficiently high frequencies ($\omega = 6\pi$), the stability boundary $\mathrm{Gr}(\mathrm{Gr}_{\mathrm{vib}})$ may have a nonuniqueness segment at $\alpha_0 = 20$–$50°$ (Fig. 1c), which is related to the suppression of vibrational instability by a sufficiently intense thermogravitational flow.

2. Three-Dimensional Perturbations. Addressing stability of a thermovibrational flow against three-dimensional perturbations in a finite-frequency vibration field, one has to use the Squire transformations, which allow one to reduce the problem of stability of plane-parallel flows against three-dimensional perturbations to the problem of planar perturbations [8]. With known threshold characteristics of flow instability against planar perturbations, the passage formulas allow one to recalculate critical convective-instability parameters for three-dimensional modes and study the competition of various types of perturbations.

We consider the amplitude problem that characterizes the behavior of normal three-dimensional perturbations in the case of a combined thermovibrational-thermogravitational flow (4). We assume that all perturbations vary periodically in the plane (x, y) of the layer and the flow velocity has three components:

$$(v_x, v_y, v_z, T, p) \sim \exp\,(ik_x x + ik_y y).$$

Here k_x and k_y are the components of the wave vector. The amplitude equations for three-dimensional perturbations, written in terms of velocity, and the boundary conditions for these equations have the form

$$\begin{gathered}
\frac{\partial v_x}{\partial t} + ik_x(\mathrm{Gr} f_1 \cos\alpha_0 + \mathrm{Gr}_{\mathrm{vib}} f_2(\omega))v_x + (\mathrm{Gr} f_1' \cos\alpha_0 + \mathrm{Gr}_{\mathrm{vib}} f_2'(\omega))v_z = -ik_x p + v_x'' - k^2 v_x + \mathrm{Gr}\theta\cos\alpha_0 + \mathrm{Gr}_{\mathrm{vib}}\theta\sin\,(\omega t),\\
\frac{\partial v_y}{\partial t} + ik_x(\mathrm{Gr} f_1 \cos\alpha_0 + \mathrm{Gr}_{\mathrm{vib}} f_2(\omega))v_y = -ik_y p + v_y'' - k^2 v_y,\\
\frac{\partial v_z}{\partial t} + ik_x(\mathrm{Gr} f_1 \cos\alpha_0 + \mathrm{Gr}_{\mathrm{vib}} f_2(\omega))v_z = -p' + v_z'' - k^2 v_z + \mathrm{Gr}\theta\sin\alpha_0,\\
\frac{\partial \theta}{\partial t} - v_z + ik_x(\mathrm{Gr} f_1 \cos\alpha_0 + \mathrm{Gr}_{\mathrm{vib}} f_2(\omega))\theta = \frac{1}{\mathrm{Pr}}(\theta'' - k^2\theta),\\
ik_x v_x + ik_y v_y + v_z' = 0, \qquad k^2 = k_x^2 + k_y^2,\\
z = \pm 1: \qquad v_x = v_y = v_z = 0, \quad \theta = 0.
\end{gathered} \tag{10}$$

The boundary-value problem for planar perturbations

$$(\bar{v}_x, \bar{v}_z, \bar{T}, \bar{p}) \sim \exp\,(i\bar{k}x)$$

can be obtained from (10), if we set $v_y = 0$ and $k_y = 0$ (we mark all unknown functions and parameters in the planar problem with a bar over these quantities).

We rewrite the amplitude problem (10) for planar perturbations as

$$\begin{gathered}
\frac{\partial \bar{v}_x}{\partial t} + i\bar{k}(\bar{\mathrm{G}}\mathrm{r} f_1 \cos\bar{\alpha}_0 + \bar{\mathrm{G}}\mathrm{r}_{\mathrm{vib}} f_2(\bar{\omega}))\bar{v}_x + (\bar{\mathrm{G}}\mathrm{r} f_1' \cos\bar{\alpha}_0 + \bar{\mathrm{G}}\mathrm{r}_{\mathrm{vib}} f_2'(\bar{\omega}))\bar{v}_z\\
= -i\bar{k}\bar{p} + \bar{v}_x'' - \bar{k}^2\bar{v}_x + \bar{\mathrm{G}}\mathrm{r}\bar{\theta}\cos\bar{\alpha}_0 + \bar{\mathrm{G}}\mathrm{r}_{\mathrm{vib}}\bar{\theta}\sin\,(\bar{\omega}t),\\
\frac{\partial \bar{v}_z}{\partial t} + i\bar{k}(\bar{\mathrm{G}}\mathrm{r} f_1 \cos\bar{\alpha}_0 + \bar{\mathrm{G}}\mathrm{r}_{\mathrm{vib}} f_2(\bar{\omega}))\bar{v}_z = -\bar{p}' + \bar{v}_z'' - \bar{k}^2\bar{v}_z + \bar{\mathrm{G}}\mathrm{r}\bar{\theta}\sin\bar{\alpha}_0,\\
\frac{\partial \bar{\theta}}{\partial t} - \bar{v}_z + i\bar{k}(\bar{\mathrm{G}}\mathrm{r} f_1 \cos\bar{\alpha}_0 + \bar{\mathrm{G}}\mathrm{r}_{\mathrm{vib}} f_2(\bar{\omega}))\bar{\theta} = \frac{1}{\bar{\mathrm{P}}\mathrm{r}}(\bar{\theta}'' - \bar{k}^2\bar{\theta}), \qquad i\bar{k}_x\bar{v}_x + \bar{v}_z' = 0,\\
z = \pm 1: \qquad \bar{v}_x = \bar{v}_z = 0, \quad \bar{\theta} = 0.
\end{gathered} \tag{11}$$

The three-dimensional problem (10) can be reduced to the two-dimensional problem (11) using the following transformations:

$$\begin{gathered}
v_z = \bar{v}_z, \quad k_x v_x + k_y v_y = \bar{k}\bar{v}_x, \quad p = \bar{p}, \quad \theta = \bar{\theta}, \quad \omega = \bar{\omega},\\
\mathrm{Pr} = \bar{\mathrm{P}}\mathrm{r}, \quad k^2 = \bar{k}^2, \quad k_x \mathrm{Gr}_{\mathrm{vib}} = \bar{k}\bar{\mathrm{G}}\mathrm{r}_{\mathrm{vib}}, \quad k_x \mathrm{Gr}\cos\alpha_0 = \bar{k}\bar{\mathrm{G}}\mathrm{r}\cos\bar{\alpha}_0,\\
\mathrm{Gr}\sin\alpha_0 = \bar{\mathrm{G}}\mathrm{r}\sin\bar{\alpha}_0.
\end{gathered}$$

Thus, in passing to three-dimensional perturbations, the vibrational and thermal Grashof numbers, the modulation frequency, the wavenumber, and the inclination angle should be transformed as follows:

$$\begin{gathered}
\mathrm{Gr}_{\mathrm{vib}} = \bar{\mathrm{G}}\mathrm{r}_{\mathrm{vib}}/a, \qquad \mathrm{Gr} = \bar{\mathrm{G}}\mathrm{r}\sqrt{\sin^2\bar{\alpha}_0 + \cos^2\bar{\alpha}_0/a},\\
\omega = \bar{\omega}, \quad k \equiv \sqrt{k_x^2 + k_y^2} = \bar{k}, \quad \tan\,\alpha_0 = a\tan\,\bar{\alpha}_0.
\end{gathered} \tag{12}$$

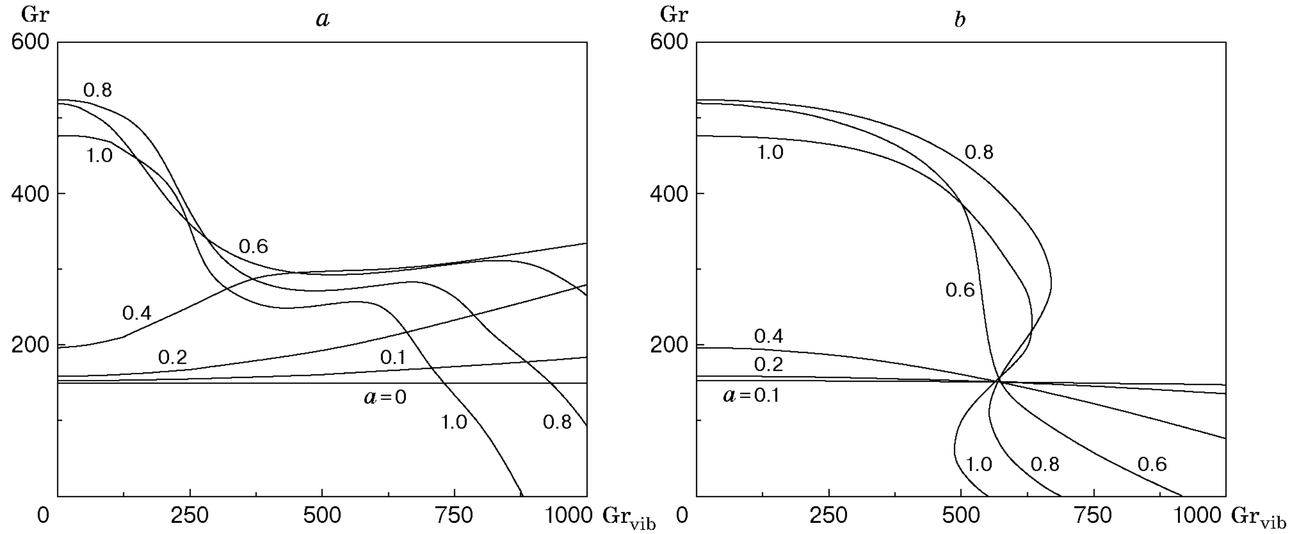

Fig. 2. Stability boundaries at $\alpha_0 = 45°$ for $\omega = 2\pi$ (a) and 6π (b).

The parameter a characterizes three-dimensional perturbations:

$$a = k_x/\sqrt{k_x^2 + k_y^2} = k_x/\bar{k}.$$

In the case of a vertical liquid layer ($\alpha_0 = 0$), it follows from (12) that

$$\bar{\alpha}_0 = 0, \quad \mathrm{Gr} = \bar{\mathrm{G}}\mathrm{r}/a, \quad \mathrm{Gr}_{\mathrm{vib}} = \bar{\mathrm{G}}\mathrm{r}_{\mathrm{vib}}/a.$$

Since the parameter a ranges in the interval $(0, 1)$, then, as in the high-frequency limit [1], we have $\mathrm{Gr} > \bar{\mathrm{G}}\mathrm{r}$ and $\mathrm{Gr}_{\mathrm{vib}} > \bar{\mathrm{G}}\mathrm{r}_{\mathrm{vib}}$, i.e., planar perturbations are more dangerous than three-dimensional ones.

In the case of a horizontally oriented layer ($\alpha_0 = \pm 90°$), the main flow has only a thermovibrational component; in this case, it follows from (12) that

$$\bar{\alpha}_0 = \pm 90°, \quad \mathrm{Gr} = \bar{\mathrm{G}}\mathrm{r}, \quad \mathrm{Gr}_{\mathrm{vib}} = \bar{\mathrm{G}}\mathrm{r}_{\mathrm{vib}}/a.$$

Thus, in the case in which the vibrations destabilize the equilibrium, the boundary of stability of the flow against three-dimensional perturbations undoubtedly lies in the region where two-dimensional perturbations are unstable, with a degeneration at the point $\mathrm{Gr}_{\mathrm{vib}} = 0$. In this case, planar and three-dimensional perturbations are equally dangerous. If vibrations stabilize the equilibrium of the liquid with respect to planar perturbations, then a competition between planar and three-dimensional modes is possible.

Figure 2 shows the boundaries of stability against three-dimensional perturbations for various values of a at $\alpha_0 = 45°$ and $\omega = 2\pi$ and 6π. At low vibration amplitudes ($\mathrm{Gr}_{\mathrm{vib}} \to 0$), as it follows from Fig. 2, a decrease in the parameter a below a certain critical value results in a decrease in the threshold Gr number; here, the most dangerous perturbations are spiral ones ($a = 0$, the instability boundary transforms into a straight line parallel to the $\mathrm{Gr}_{\mathrm{vib}}$-axis). At a low intensity of the thermogravitational flow ($\mathrm{Gr} \to 0$), a decrease in a results in an increase of the critical vibration amplitude $\mathrm{Gr}_{\mathrm{vib}}$ corresponding to the threshold of stability; hence, the most dangerous perturbations here are planar perturbations ($a = 1$).

The actual scenario of evolution of the stability boundary during the passage from planar ($a = 1$) to spiral perturbations ($a = 0$) depends on vibration frequency. At $\omega = 2\pi$ (Fig. 2a), the dependence $\mathrm{Gr} = f(\mathrm{Gr}_{\mathrm{vib}})$ for planar perturbations displays a local maximum in the region of $\mathrm{Gr}_{\mathrm{vib}} \approx 600$. With decreasing a, this maximum is shifted toward higher values of $\mathrm{Gr}_{\mathrm{vib}}$ and then becomes an absolute one, displaced to infinity. At $\omega = 6\pi$ (Fig. 2b), the dependence $\mathrm{Gr} = f(\mathrm{Gr}_{\mathrm{vib}})$ for all values of a has only one maximum at $\mathrm{Gr}_{\mathrm{vib}} = 0$. At $a = 0.2$ and 0.4, the dependences $\mathrm{Gr} = f(\mathrm{Gr}_{\mathrm{vib}})$ are parabolic; in this case, vibrations destabilize the equilibrium with respect to these three-dimensional modes.

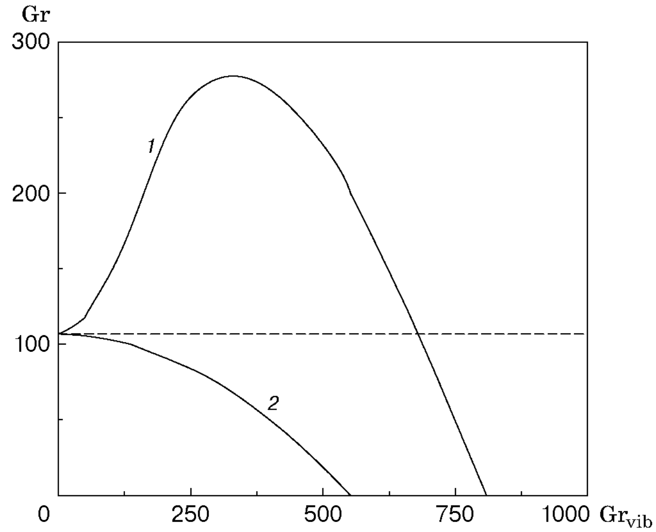

Fig. 3. Stability boundaries for a horizontal layer ($\alpha_0 = 90°$) for $\omega = \pi$ (1) and 8π (2); the dashed curve refers to the stability boundary for the spiral mode $\mathrm{Gr}_{\mathrm{sp}}$.

The stability boundary for spiral perturbations ($a = 0$) can be found from (10) with $k_x = 0$:

$$\frac{\partial v_x}{\partial t} = v_x'' - k_y^2 v_x + \mathrm{Gr}\theta\cos\alpha_0 + \mathrm{Gr}_{\mathrm{vib}}\theta\sin(\omega t),$$

$$\frac{\partial v_y}{\partial t} = -ik_y p + v_y'' - k_y^2 v_y, \qquad \frac{\partial v_z}{\partial t} = -p' + v_z'' - k_y^2 v_z + \mathrm{Gr}\theta\sin\alpha_0,$$

$$\frac{\partial \theta}{\partial t} - v_z = \frac{1}{\mathrm{Pr}}(\theta'' - k_y^2\theta), \quad ik_y v_y + v_z' = 0, \tag{13}$$

$$z = \pm 1: \qquad v_x = v_y = v_z = 0, \quad \theta = 0.$$

The critical Grashof number for spiral perturbations $\mathrm{Gr}_{\mathrm{sp}}$ is independent of vibration amplitude and frequency. The latter parameters affect only the intensity of the disturbed flow along convective rollers. An analysis of Eqs. (13) shows that the Grashof number $\mathrm{Gr}_{\mathrm{sp}}$ corresponds to the case of a vibration-free thermogravitational flow: $\mathrm{Gr} = 106.7/(\mathrm{Pr}\sin\alpha_0)$ [8]. The region where the combined thermogravitational-thermovibrational flow is absolutely stable (Fig. 2) lies between the coordinate axes, the horizontal boundary for spiral perturbations ($a = 0$), and the right-hand boundary for planar perturbations ($a = 1$).

In the case of a horizontally oriented layer ($\alpha_0 = 90°$), the stability boundary for spiral and planar perturbations are shown in Fig. 3. Curves 1 and 2 show the results gained in [7]. It follows from Fig. 3 that the stability threshold for $\omega = 8\pi$ is fully determined by the behavior of planar perturbations. In the case of $\omega = \pi$, in the range of vibrational Grashof numbers $0 < \mathrm{Gr}_{\mathrm{vib}} < 683.8$, the instability boundary ($\mathrm{Gr} = 106.7$) is determined by spiral perturbations; at higher vibration amplitudes ($\mathrm{Gr}_{\mathrm{vib}} > 683.8$), planar perturbations grow in value on the curve $\mathrm{Gr} = f(\mathrm{Gr}_{\mathrm{vib}})$.

Summary. Using the Floquet theory, we have considered the problem of instability of a uniform flow of an inclined liquid layer under the action of a gravity force and streamwise finite-frequency perturbations. In the case of planar perturbations, either destabilization or stabilization of the ground state is possible depending on the characteristics of the parametric action. In the examined range of inclination angles of the layer, vibration amplitudes, and vibration frequencies, flow instability is caused by in-phase perturbations. For the case of a periodical thermovibrational flow, transformations are obtained that relate characteristics of planar and three-dimensional perturbations. It is shown that, generally, stability of the main flow can be violated either by spiral or by planar perturbations. The critical characteristics of spiral perturbations are independent of vibration amplitude and frequency.

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